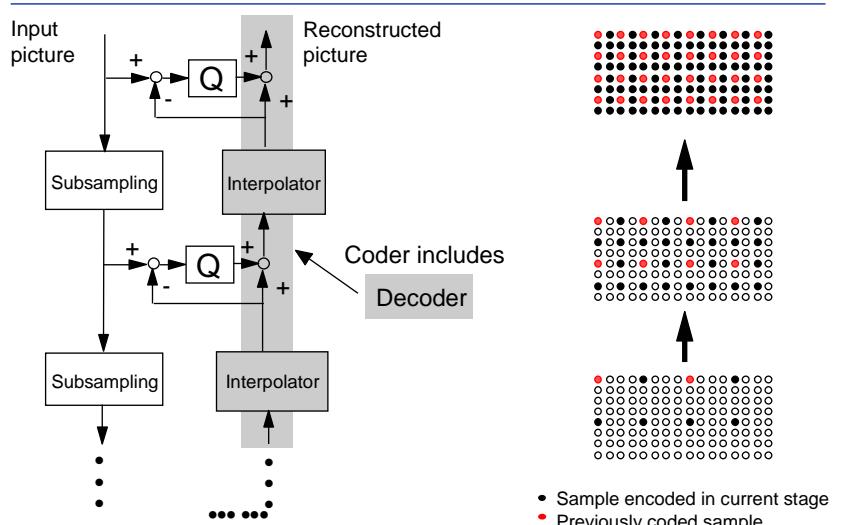


Pyramid Coding and Subband Coding

- Predictive pyramids
- Transform pyramids
- Subband coding
- Perfect reconstruction filter banks
- Quadrature mirror filter banks
- Octave band splitting
- Transform coding as a special case of subband coding



Interpolation Error Coding, I



Interpolation Error Coding, II



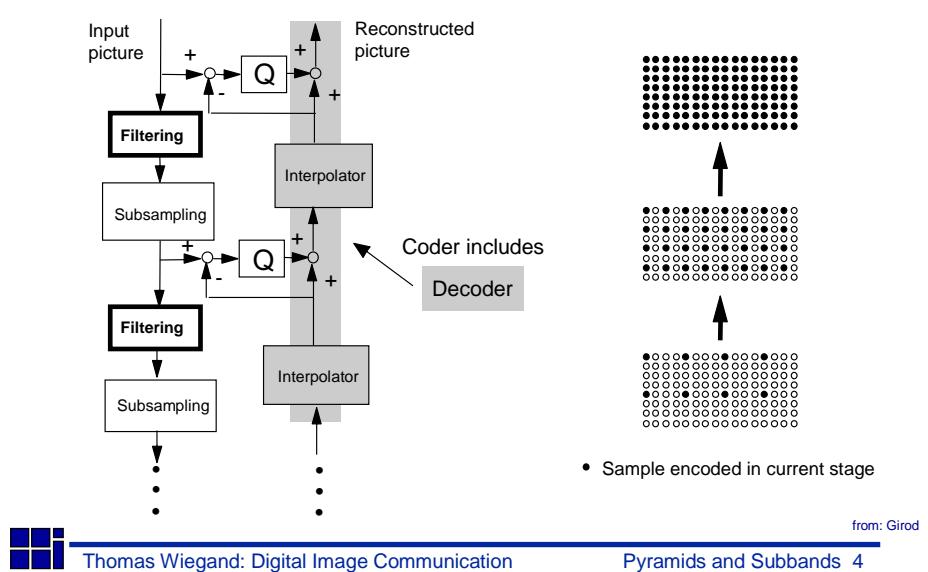
from: Girod



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Pyramids and Subbands 3

Predictive Pyramid, I



from: Girod



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Pyramids and Subbands 4

Predictive Pyramid, II

Number of samples to be encoded =

$$(1 + \frac{1}{N} + \frac{1}{N^2} + \dots) = \frac{N}{N-1}$$

↑
Subsampling factor

x number of original image samples



Predictive Pyramid, III



transmitted signals



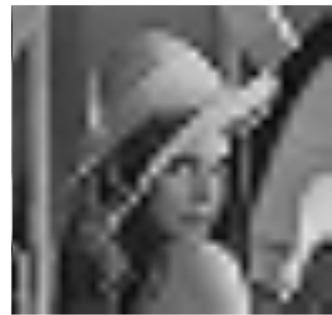
Comparison: Interpolation Error Coding vs. Pyramid, I

- Resolution layer #0 (lowest resolution),
interpolated to original size for display

Interpolation Error Coding



Pyramid



from: Girod



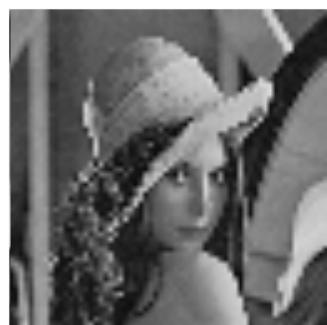
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Pyramids and Subbands 7

Comparison: Interpolation Error Coding vs. Pyramid, II

- Resolution layer #1,
interpolated to original size for display

Interpolation Error Coding



Pyramid



from: Girod



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Pyramids and Subbands 8

Comparison: Interpolation Error Coding vs. Pyramid, III

- Resolution layer #2 ,
interpolated to original size for display

Interpolation Error Coding



Pyramid



from: Girod



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Pyramids and Subbands 9

Comparison: Interpolation Error Coding vs. Pyramid, IV

- Resolution layer #3

Interpolation Error Coding



Pyramid



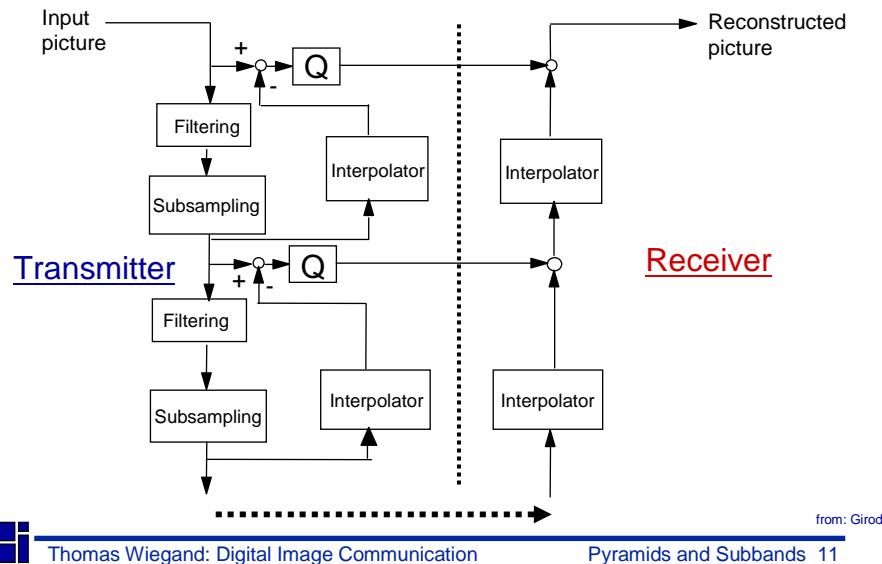
from: Girod



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Pyramids and Subbands 10

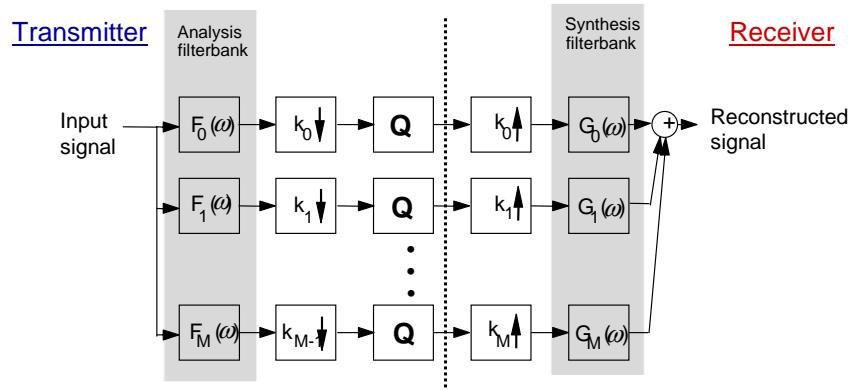
Transform Pyramid Coding



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Pyramids and Subbands 11

Subband Coding



- Number of degrees of freedom is preserved: $\frac{1}{K_0} + \frac{1}{K_1} + \dots + \frac{1}{K_M} = 1$
- Perfect reconstruction filterbank required

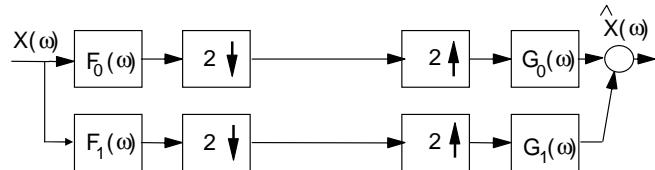


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Pyramids and Subbands 12

from: Girod

Two-Channel Filterbank



$$\begin{aligned}\hat{X}(\omega) = & \frac{1}{2} [F_0(\omega) G_0(\omega) + F_1(\omega) G_1(\omega)] X(\omega) \\ & + \frac{1}{2} [F_0(\omega + \pi) G_0(\omega) + F_1(\omega + \pi) G_1(\omega)] X(\omega + \pi)\end{aligned}$$

Aliasing

- Aliasing cancellation if :

$$\begin{aligned}G_0(\omega) &= F_1(\omega + \pi) \\ -G_1(\omega) &= F_0(\omega + \pi)\end{aligned}$$

from: Girod



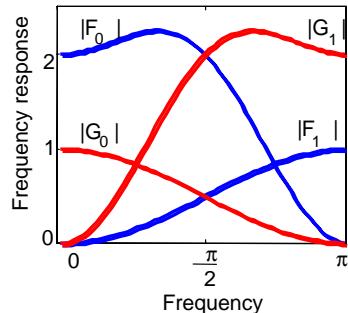
Example : Two-Channel Filterbank with Perfect Reconstruction

- Analysis filter impulse responses:

- Lowpass band $\frac{1}{4}(-1, +2, +6, +2, -1)$
- Highpass band $\frac{1}{4}(+1, -2, +1)$

- Synthesis filter impulse responses:

- Lowpass band: $\frac{1}{4}(+1, +2, +1)$
- Highpass band: $\frac{1}{4}(+1, +2, -6, +2, +1)$



from: Girod

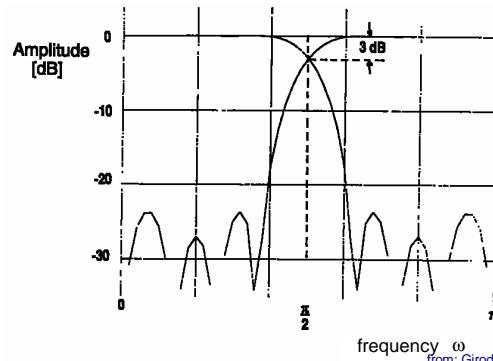
Quadrature Mirror Filters (QMF)

- QMFs achieve aliasing cancellation by choosing

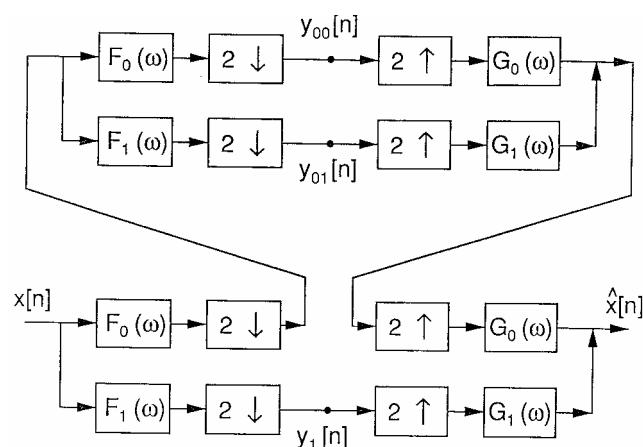
$$\begin{aligned} F_i(\omega) &= F_o(\omega + \pi) \\ &= -G_i(\omega) = G_o(\omega + \pi) \end{aligned}$$

- Highpass band is the mirror image of the lowpass band in the frequency domain

Example:
16-tap QMF filterbank:

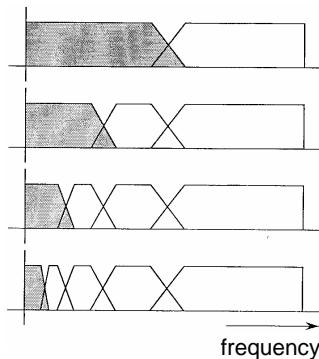


Cascaded Analysis / Synthesis Filterbanks



Octave Band Splitting

- Recursive application of a two-band filter bank to the lowpass band of the previous stage yields octave band splitting:

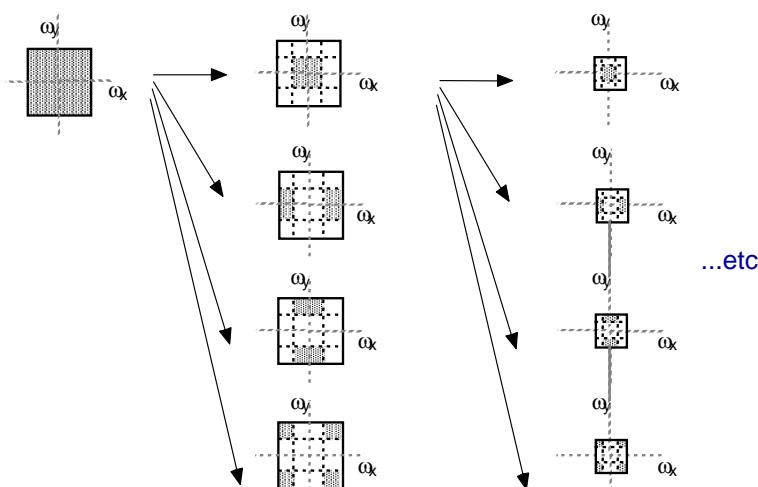


- Same concept, but derived from wavelet theory:
dyadic wavelet decomposition

from: Girod



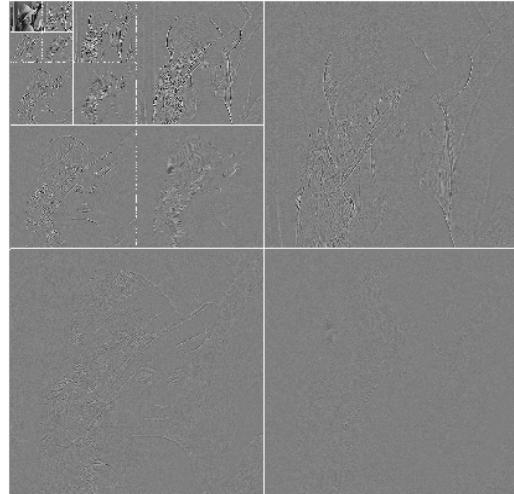
Separable 2D Filterbank, I



from: Girod



Separable 2D Filterbank, II



from: Girod



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Pyramids and Subbands 19

Subband Coding vs. Transform Coding, I

- Transform coding is a special case of subband coding with:
 - Number of bands = order of transform N
 - Subsampling factor $K = N$
 - Length of impulse responses of analysis/synthesis filters $\leq N$
- Filters used in subband coders are not in general orthogonal.

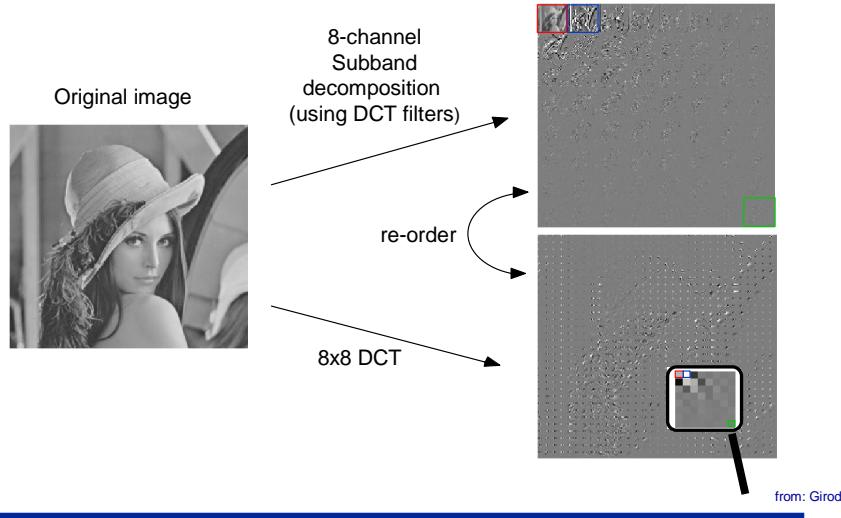
from: Girod



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Pyramids and Subbands 20

Subband Coding vs. Transform Coding, II



Summary: Pyramid Coding and Subband Coding

- Resolution pyramids with subsampling 2:1 horizontally and vertically
- Predictive pyramids: quantization error feedback („closed loop“)
- Transform pyramids: no quantization error feedback („open loop“)
- Pyramids: overcomplete representation of the image
- Application of pyramids: coarse-to-fine transmission, unequal error protection of resolution layers
- Subband coding: number of samples not increased
- Quadrature mirror filters: aliasing cancellation
- Transform coding is subband coding with non-overlapping impulse responses

