
Predictive Coding

- Prediction
- Prediction in Images
- Principle of Differential Pulse Code Modulation (DPCM)
- DPCM and entropy-constrained scalar quantization
- DPCM and transmission errors
- Adaptive intra-interframe DPCM
- Conditional Replenishment

Prediction

Prediction is difficult – especially for the future.

Mark Twain

- Prediction: *Statistical estimation procedure where future random variables are estimated/predicted from past and present observable random variables.*
- Prediction from previous samples: $\hat{S}_0 = f(S_1, S_2, \dots, S_N) = f(\mathbf{S})$
- Optimization criterion

$$E\{(S_0 - \hat{S}_0)^2\} = E\{[S_0 - f(S_1, S_2, \dots, S_N)]^2\} \rightarrow \min$$

- Optimum predictor:

$$\hat{S}_0 = E\{S_0 | (S_1, S_2, \dots, S_N)\}$$

Structure

- The optimum predictor $\hat{S}_0 = E\{S_0 | (S_1, S_2, \dots, S_N)\}$ can be stored in a table (Pixels: 8 bit \rightarrow size 2^{8N})
- Optimal linear prediction (zero mean, Gaussian RVs)

$$\hat{S}_0 = a_1 S_1 + a_2 S_2 + \dots + a_N S_N = \mathbf{a}^t \mathbf{S}$$

- Optimization criterion

$$E\{(S_0 - \hat{S}_0)^2\} = E\{(S_0 - \mathbf{a}^t \mathbf{S})^2\}$$

- Optimum linear predictor is solution of

$$\mathbf{a}^t \mathbf{R}_S = E\{S_0 \mathbf{S}^t\}$$

- In case $\mathbf{R}_S = E\{\mathbf{S} \mathbf{S}^t\}$ is invertible

$$\mathbf{a} = \mathbf{R}_S^{-1} E\{S_0 \mathbf{S}\}$$

Prediction in Images: Intra-frame Prediction

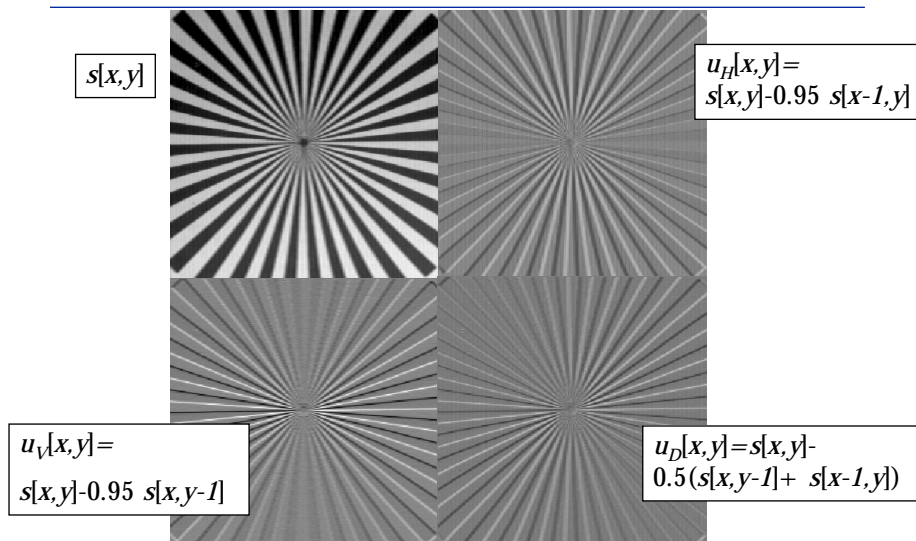
- Past and present observable random variables are prior scanned pixels within that image
- When scanning from upper left corner to lower right corner:

B	C	D
A	X	

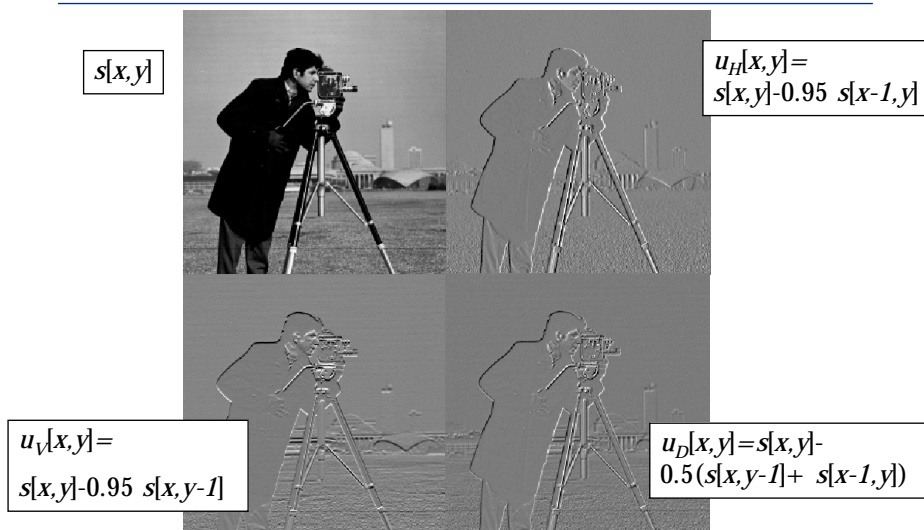
- 1-D Horizontal prediction: A only
- 1-D Vertical prediction: C only
- Improvements for 2-D approaches (requires line store)

$$\hat{s}(x, y) = \sum_{\substack{p=-P_1 \\ (p,q) \neq (0,0)}}^{P_2} \sum_{q=0}^{Q_1} a(p, q) \cdot s(x - p, y - q)$$

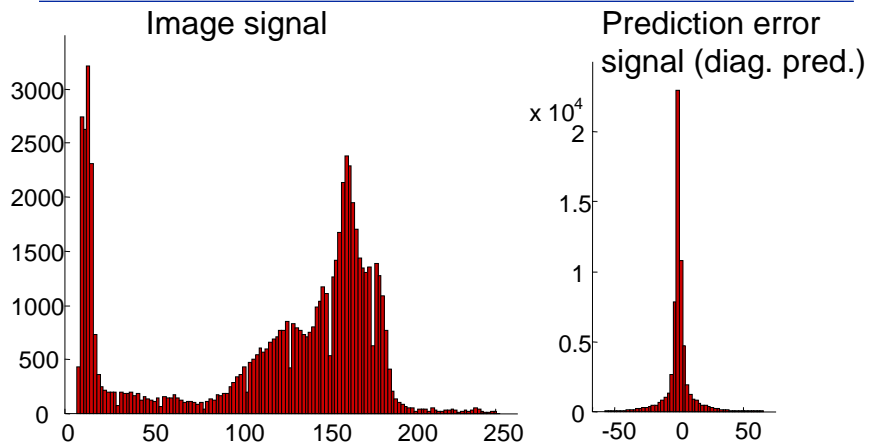
Prediction Example: Test Pattern



Prediction Example: Cameraman



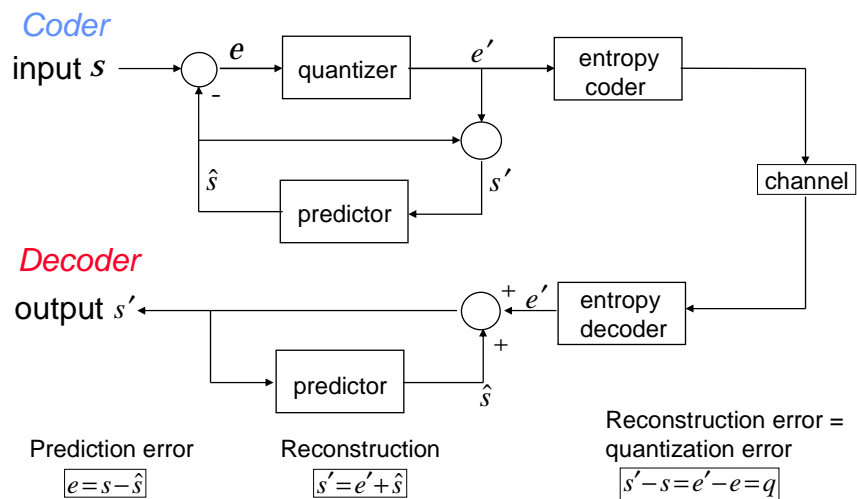
Change of Histograms: Cameraman



Can we use prediction for compression ?

Yes, if we reproduce the prediction signal at the decoder

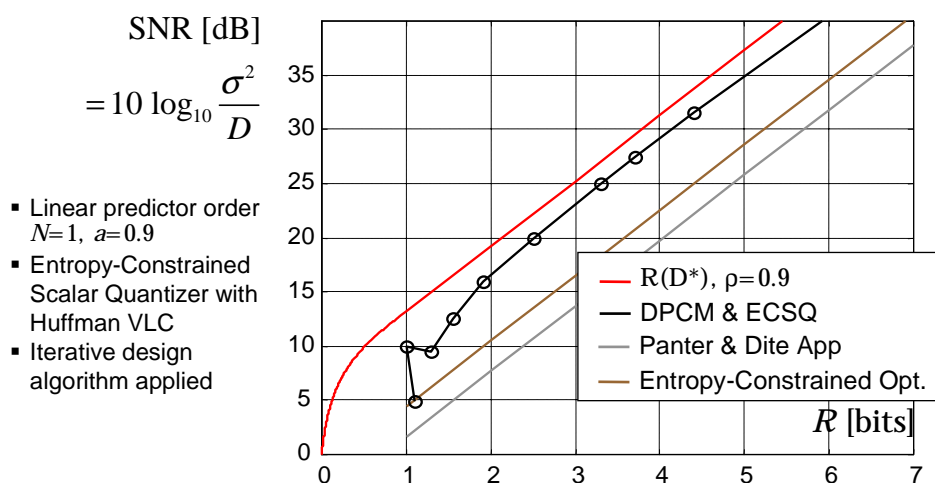
Differential Pulse Code Modulation



DPCM and Quantization

- Prediction is based on quantized samples
- Stability problems for large quantization errors
- Prediction shapes error signal (typical pdfs: Laplacian, generalized Gaussian)
- Simple and efficient: combine with entropy-constrained scalar quantization
- Higher gains: Combine with block entropy coding
- Use a switched predictor
 - Forward adaptation (side information)
 - Backward adaptation (error resilience, accuracy)
- DPCM can also be conducted for vectors
 - Predict vectors (with side information)
 - Quantize prediction error vectors

Comparison for Gauss-Markov Source: $r=0.9$



DPCM with Entropy-Constrained Scalar Quantization

Example: Lena, 8 b/p



$K=511, H=4.79 \text{ b/p}$ $K=15, H=1.98 \text{ b/p}$ $K=3, H=0.88 \text{ b/p}$
 K ...number of reconstruction levels, H ...entropy

from: Ohm

Transmission Errors in a DPCM System

- For a linear DPCM decoder, the transmission error response is superimposed to the reconstructed signal S'
- For a stable DPCM decoder, the transmission error response decays
- Finite word-length effects in the decoder can lead to residual errors that do not decay (e.g., limit cycles)

from: Girod

Transmission Errors in a DPCM System II

Example: Lena, 3 b/p (fixed code word length)



Error rate $p=10^{-3}$.

1D pred., hor. $a_H=0.95$

1D pred., ver. $a_V=0.95$

2D pred.*, $a_H=a_V=0.5$

from: Ohm

Inter-frame Coding of Video Signals

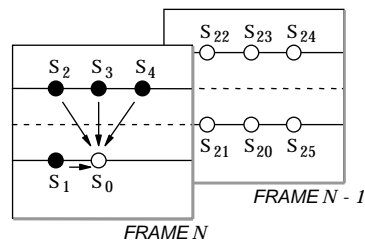
- Inter-frame coding exploits:
 - Similarity of temporally successive pictures
 - Temporal properties of human vision
- Important inter-frame coding methods:
 - Adaptive intra/inter-frame coding
 - Conditional replenishment
 - Motion-compensating prediction (in Hybrid Video Coding)
 - Motion-compensating interpolation

from: Girod

Principle of Adaptive Intra/Inter-Frame DPCM

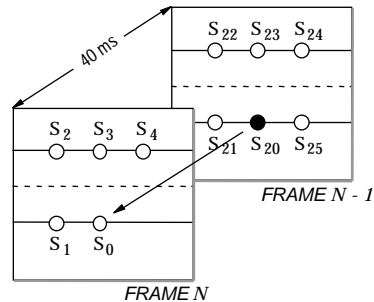
Predictor is switched between two states:
for moving or changed areas.

Intra-frame prediction
for moving or changed areas.



$$\hat{S} = a_1 \cdot S'_1 + a_2 \cdot S'_2 + a_3 \cdot S'_3 + a_4 \cdot S'_4$$

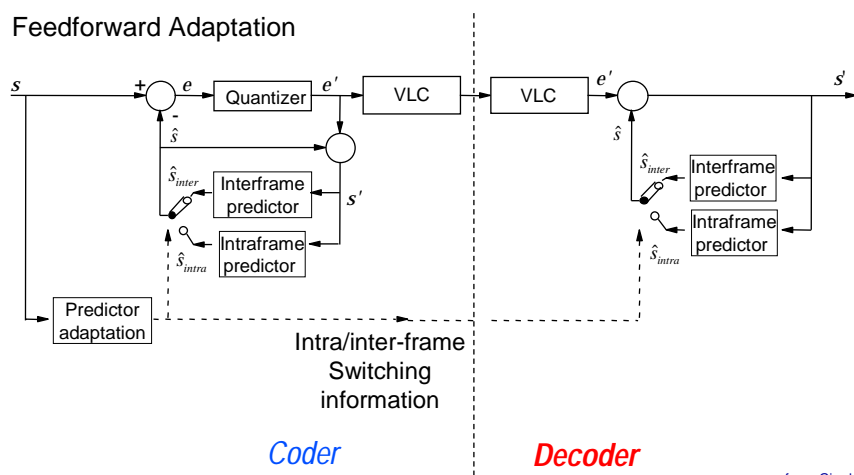
Inter-frame prediction (previous frame
prediction) for still areas of the picture.



$$\hat{S}_{inter} = S'_{20} \quad \text{from: Girod}$$

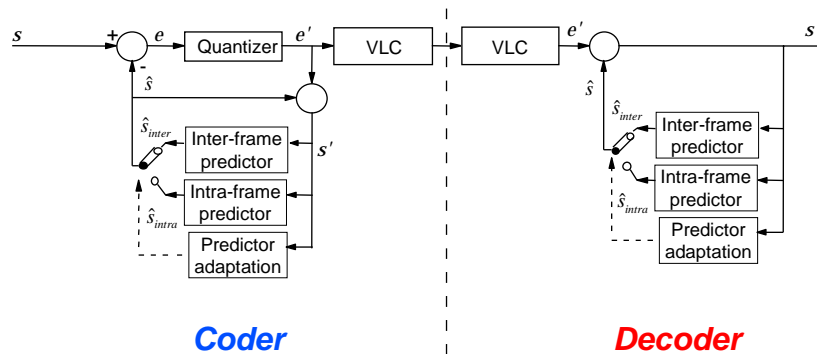
Intra/Inter-Frame DPCM: Adaptation Strategies, I

Feedforward Adaptation



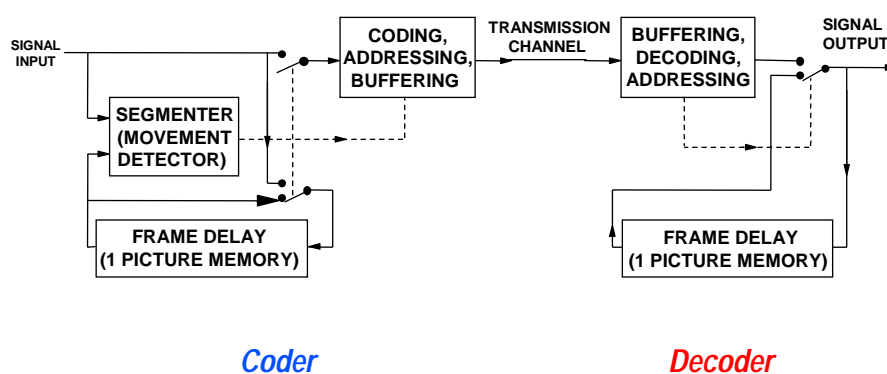
Intra/Inter-Frame DPCM: Adaptation Strategies, II

Feedback Adaptation



from: Girod

Principle of a Conditional Replenishment Coder

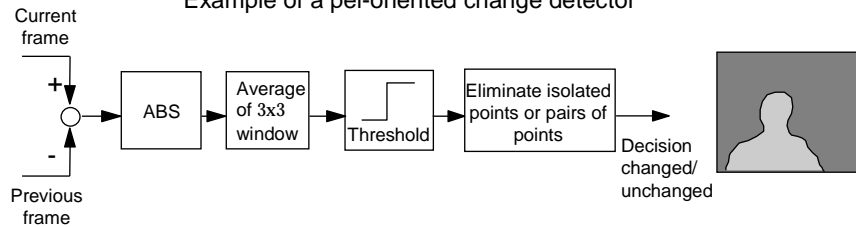


- Still areas: repeat from frame store
- Moving areas: transmit address and waveform

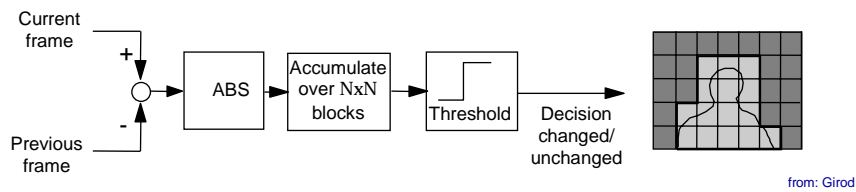
from: Girod

Change Detection

Example of a pel-oriented change detector

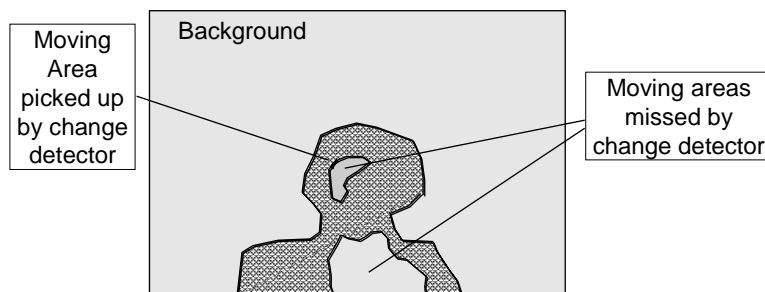


Example of a block-oriented change detector



The "Dirty Window" Effect

Conditional replenishment scheme with change detection threshold set too high leads to the subjective impression of looking through a dirty window.



from: Girod

Summary

- Prediction: Estimation of random variable from past or present observable random variables
- Optimal prediction
- Optimal linear prediction
- Prediction in images: 1-D vs. 2-D prediction
- DPCM: Prediction from previously coded/transmitted samples (known at coder and decoder)
- DPCM and quantization
- DPCM and transmission errors
- Adaptive Intra/Inter-frame DPCM: forward adaptation vs. backward adaptation
- Conditional Replenishment: Only changed areas of image are transmitted