Predictive Coding

- Prediction
- Prediction in Images
- Principle of Differential Pulse Code Modulation (DPCM)
- DPCM and entropy-constrained scalar quantization
- DPCM and transmission errors
- Adaptive intra-interframe DPCM
- Conditional Replenishment

Prediction

Prediction is difficult – especially for the future.
Mark Twain

- Prediction: *Statistical estimation procedure where future random variables are estimated/predicted from past and present observable random variables.*

- Prediction from previous samples: $\hat{S}_0 = f(S_1, S_2, \ldots, S_N) = f(\mathbf{S})$
- Optimization criterion

$$E\left\{\left(S_0 - \hat{S}_0\right)^2\right\} = E\left\{\left[S_0 - f(S_1, S_2, \ldots, S_N)\right]^2\right\} \rightarrow \min$$

- Optimum predictor:

$$\hat{S}_0 = E\left\{S_0 \mid (S_1, S_2, \ldots, S_N)\right\}$$

Structure

- The optimum predictor $\hat{S}_0 = E\{S_0 \,|\, (S_1, S_2, \ldots, S_N)\}$ can be stored in a table (Pixels: 8 bit ➔ size 2^{8N})
- Optimal linear prediction (zero mean, Gaussian RVs)

$$\hat{S}_0 = a_1 S_1 + a_2 S_2 + \ldots + a_N S_N = \mathbf{a}^t \mathbf{S}$$

- Optimization criterion

$$E\left\{\left(S_0 - \hat{S}_0\right)^2\right\} = E\left\{\left(S_0 - \mathbf{a}^t \mathbf{S}\right)^2\right\}$$

- Optimum linear predictor is solution of

$$\mathbf{a}^t \mathbf{R}_\mathbf{S} = E\left\{S_0 \mathbf{S}^t\right\}$$

- In case $\mathbf{R}_\mathbf{S} = E(\mathbf{S}\mathbf{S}^t)$ is invertible

$$\mathbf{a} = \mathbf{R}_\mathbf{S}^{-1} E\left\{S_0 \mathbf{S}\right\}$$

Prediction in Images: Intra-frame Prediction

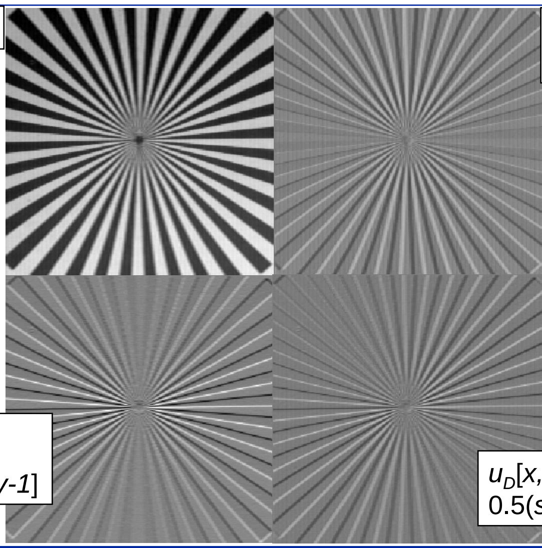
- Past and present observable random variables are prior scanned pixels within that image
- When scanning from upper left corner to lower right corner:

B	C	D
A	X	

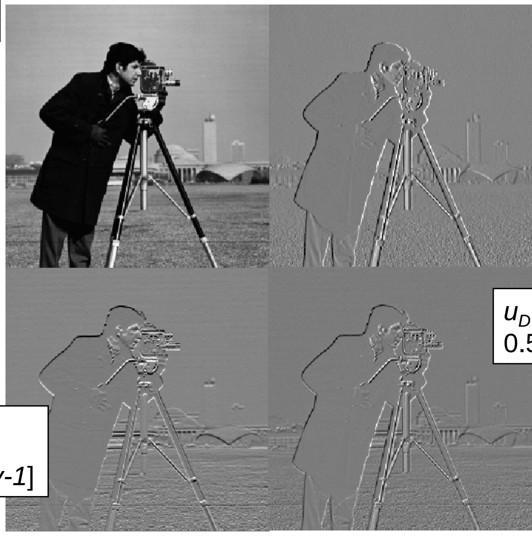
- 1-D Horizontal prediction: A only
- 1-D Vertical prediction: C only
- Improvements for 2-D approaches (requires line store)

$$\hat{s}(x,y) = \sum_{\substack{p=-P_1 \\ (p,q)\neq(0,0)}}^{P_2} \sum_{q=0}^{Q} a(p,q) \cdot s(x-p, y-q)$$

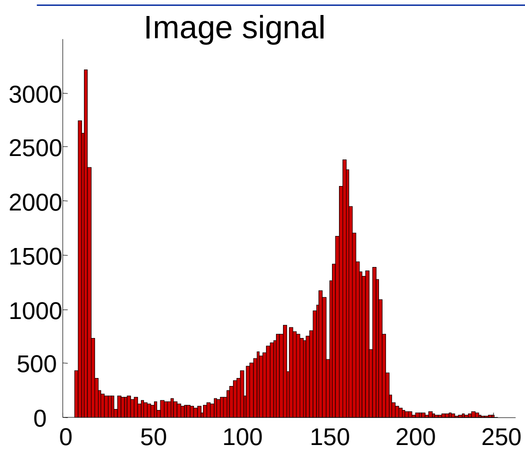
Prediction Example: Test Pattern

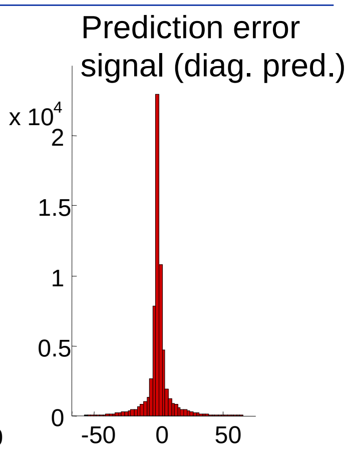


Prediction Example: Cameraman



Change of Histograms: Cameraman

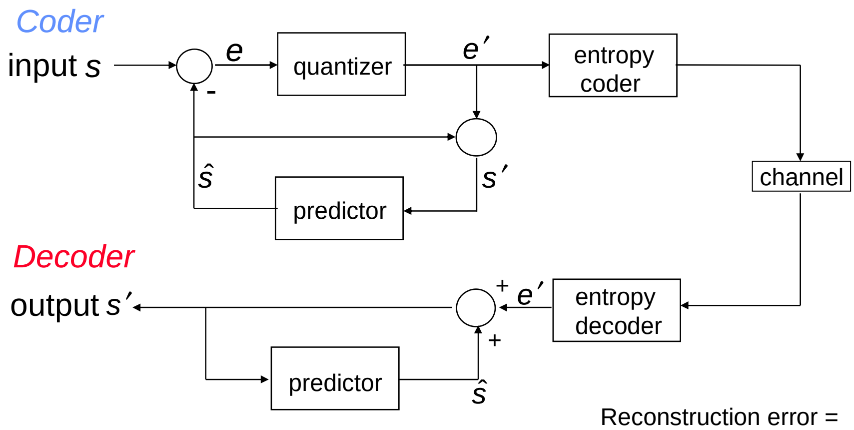




Can we use prediction for compression ?

Yes, if we reproduce the prediction signal at the decoder

Differential Pulse Code Modulation



Prediction error	Reconstruction	Reconstruction error = quantization error
$e = s - \hat{s}$	$s' = e' + \hat{s}$	$s' - s = e' - e = q$

DPCM and Quantization

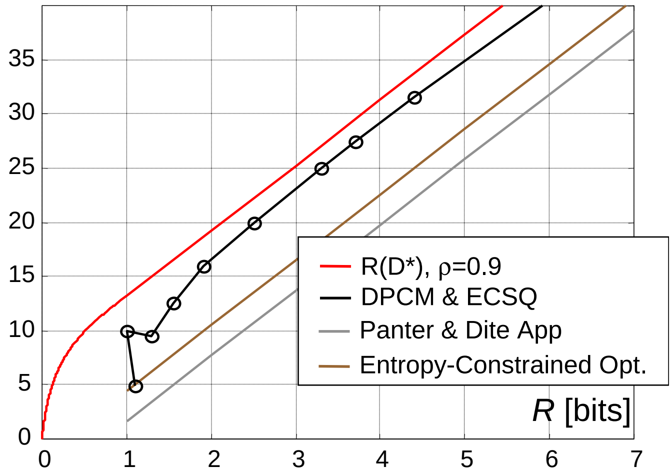
- Prediction is based on quantized samples
- Stability problems for large quantization errors
- Prediction shapes error signal (typical pdfs: Laplacian, generalized Gaussian)
- Simple and efficient: combine with entropy-constrained scalar quantization
- Higher gains: Combine with block entropy coding
- Use a switched predictor
  - Forward adaptation (side information)
  - Backward adaptation (error resilience, accuracy)
- DPCM can also be conducted for vectors
  - Predict vectors (with side information)
  - Quantize prediction error vectors

Comparison for Gauss-Markov Source: ρ=0.9

SNR [dB]

$$= 10\log_{10} \frac{\sigma^2}{D}$$

- Linear predictor order N=1, a=0.9
- Entropy-Constrained Scalar Quantizer with Huffman VLC
- Iterative design algorithm applied



DPCM with Entropy-Constrained Scalar Quantization

Example: Lena, 8 *b/p*

K=511, *H*=4.79 *b*/*p*   *K*=15, *H*=1.98 *b*/*p*   *K*=3, *H*=0.88 *b*/*p*
K...number of reconstruction levels, *H*...entropy

from: Ohm

Transmission Errors in a DPCM System

- For a linear DPCM decoder, the transmission error response is superimposed to the reconstructed signal *S'*
- For a stable DPCM decoder, the transmission error response decays
- Finite word-length effects in the decoder can lead to residual errors that do not decay (e.g., limit cycles)

from: Girod

Transmission Errors in a DPCM System II

Example: Lena, 3 *b/p* (fixed code word length)

Error rate p=10^{-3}.
1D pred., hor. a_H=0.95 1D pred., ver. a_V=0.95 2D pred.*,a_H=a_V=0. 5

from: Ohm

Inter-frame Coding of Video Signals

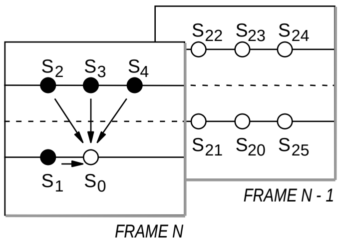
- Inter-frame coding exploits:
  - Similarity of temporally successive pictures
  - Temporal properties of human vision
- Important inter-frame coding methods:
  - Adaptive intra/inter-frame coding
  - Conditional replenishment
  - Motion-compensating prediction (in Hybrid Video Coding)
  - Motion-compensating interpolation

from: Girod

Principle of Adaptive Intra/Inter-Frame DPCM

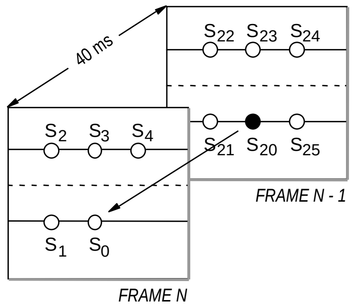
Predictor is switched between two states:
for moving or changed areas.

Intra-frame prediction
for moving or changed areas.



$$\hat{S} = a_1 \cdot S_1' + a_2 \cdot S_2' + a_3 \cdot S_3' + a_4 S_4'$$

Inter-frame prediction (previous frame prediction) for still areas of the picture.

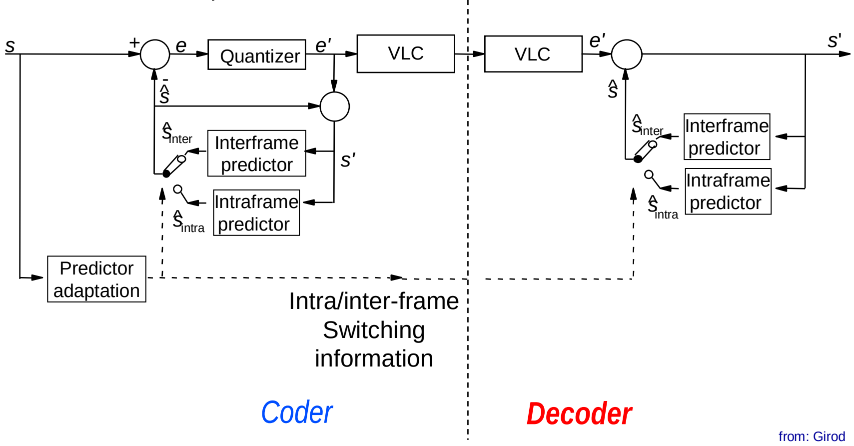


$$\hat{S}_{inter} = S'_{20}$$

from: Girod

Intra/Inter-Frame DPCM: Adaptation Strategies, I

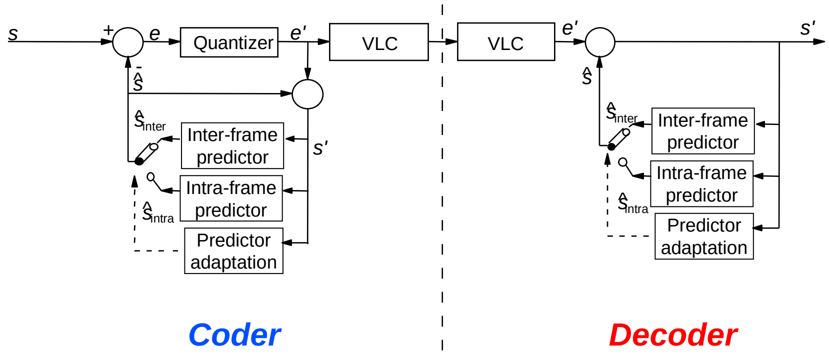
Feedforward Adaptation



from: Girod

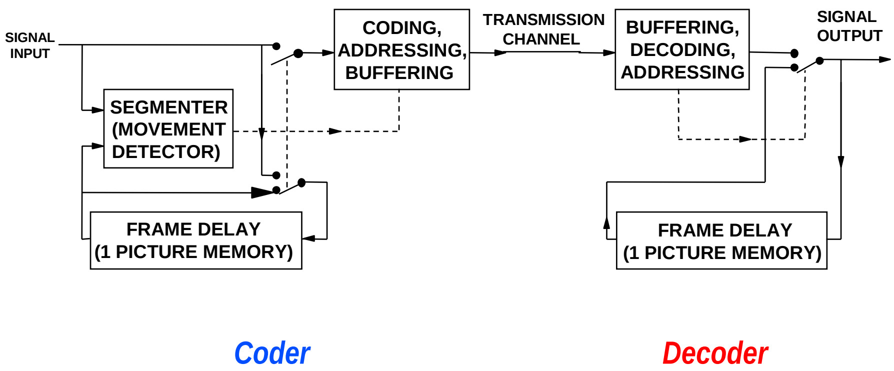
Intra/Inter-Frame DPCM: Adaptation Strategies, II

Feedback Adaptation



from: Girod

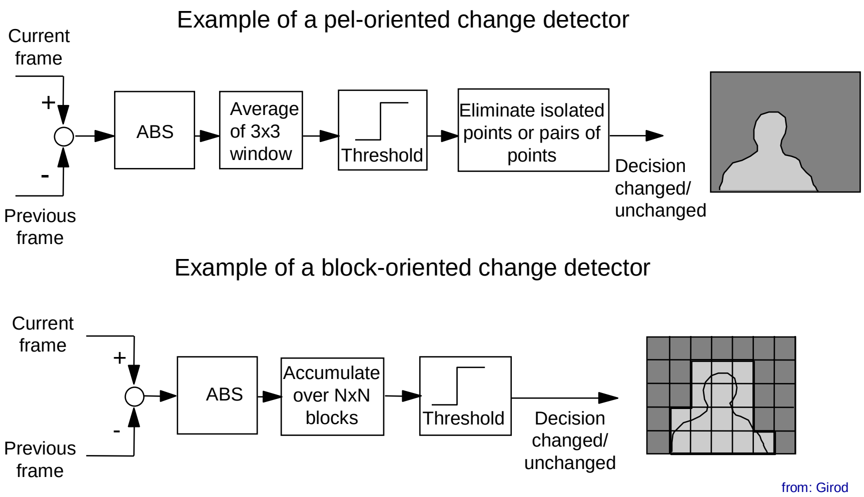
Principle of a Conditional Replenishment Coder



- Still areas: repeat from frame store
- Moving areas: transmit address and waveform

from: Girod

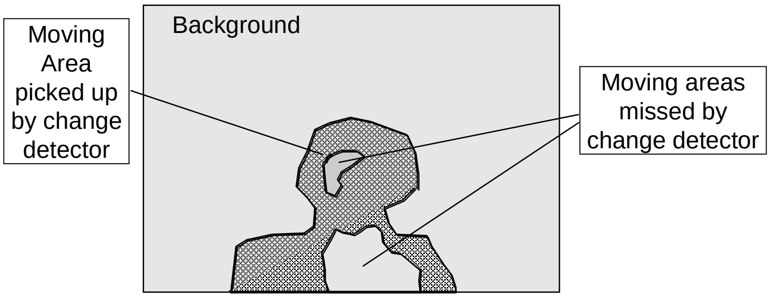
Change Detection



from: Girod

The "Dirty Window" Effect

Conditional replenishment scheme with change detection threshold set too high leads to the subjective impression of looking through a dirty window.



from: Girod

Summary

- Prediction: Estimation of random variable from past or present observable random variables
- Optimal prediction
- Optimal linear prediction
- Prediction in images: 1-D vs. 2-D prediction
- DPCM: Prediction from previously coded/transmitted samples (known at coder and decoder)
- DPCM and quantization
- DPCM and transmission errors
- Adaptive Intra/Inter-frame DPCM: forward adaptation vs. backward adaptation
- Conditional Replenishment: Only changed areas of image are transmitted