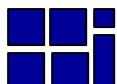

Predictive Coding

- Prediction
- Prediction in Images
- Principle of Differential Pulse Code Modulation (DPCM)
- DPCM and entropy-constrained scalar quantization
- DPCM and transmission errors
- Adaptive intra-interframe DPCM
- Conditional Replenishment



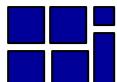
Prediction

Prediction is difficult – especially for the future.

Mark Twain

- Prediction: *Statistical estimation procedure where future random variables are estimated/predicted from past and present observable random variables.*
- Prediction from previous samples: $\hat{S}_0 = f(S_1, S_2, \dots, S_N) = f(\mathbf{S})$
- Optimization criterion
$$E\{(S_0 - \hat{S}_0)^2\} = E\{[S_0 - f(S_1, S_2, \dots, S_N)]^2\} \rightarrow \min$$
- Optimum predictor:

$$\hat{S}_0 = E\{S_0 | (S_1, S_2, \dots, S_N)\}$$



Structure

- The optimum predictor $\hat{S}_0 = E\{S_0 | (S_1, S_2, \dots, S_N)\}$ can be stored in a table (Pixels: 8 bit → size 2^{8N})
- Optimal linear prediction (zero mean, Gaussian RVs)

$$\hat{S}_0 = a_1 S_1 + a_2 S_2 + \dots + a_N S_N = \mathbf{a}^t \mathbf{S}$$

- Optimization criterion

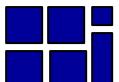
$$E\{(S_0 - \hat{S}_0)^2\} = E\{(S_0 - \mathbf{a}^t \mathbf{S})^2\}$$

- Optimum linear predictor is solution of

$$\mathbf{a}^t \mathbf{R}_s = E\{S_0 \mathbf{S}^t\}$$

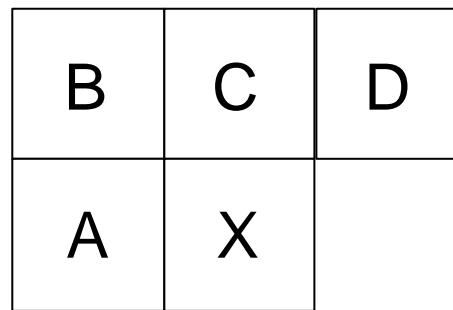
- In case $\mathbf{R}_s = E(\mathbf{S}\mathbf{S}^t)$ is invertible

$$\mathbf{a} = \mathbf{R}_s^{-1} E\{S_0 \mathbf{S}\}$$



Prediction in Images: Intra-frame Prediction

- Past and present observable random variables are prior scanned pixels within that image
- When scanning from upper left corner to lower right corner:



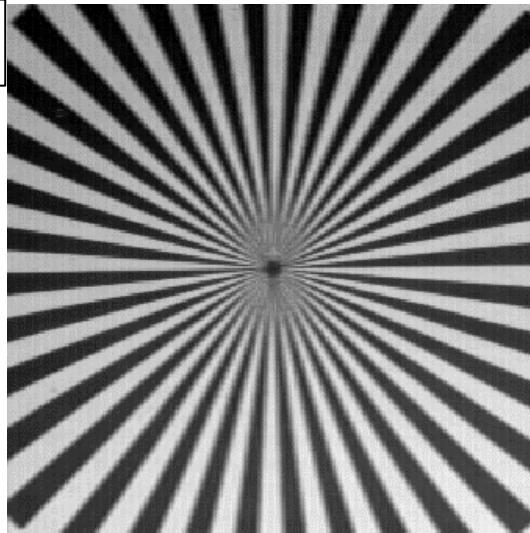
- 1-D Horizontal prediction: A only
- 1-D Vertical prediction: C only
- Improvements for 2-D approaches (requires line store)

$$\hat{s}(x,y) = \sum_{\substack{p=-P_1 \\ (p,q)\neq(0,0)}}^{P_2} \sum_{q=0}^Q a(p,q) \cdot s(x-p, y-q)$$



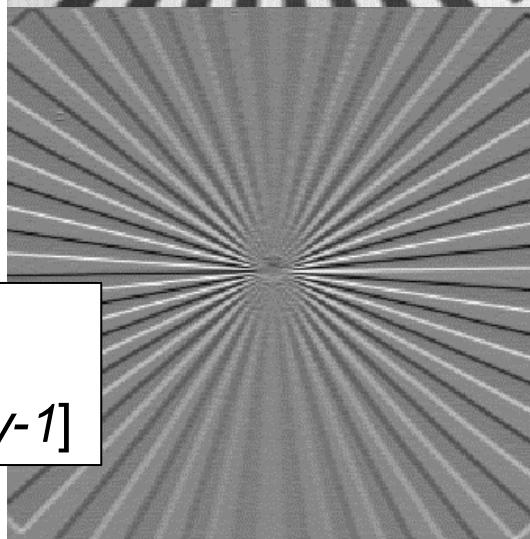
Prediction Example: Test Pattern

$s[x,y]$



$$u_H[x,y] = s[x,y] - 0.95 s[x-1,y]$$

$$u_V[x,y] = s[x,y] - 0.95 s[x,y-1]$$



$$u_D[x,y] = s[x,y] - 0.5(s[x,y-1] + s[x-1,y])$$



Prediction Example: Cameraman

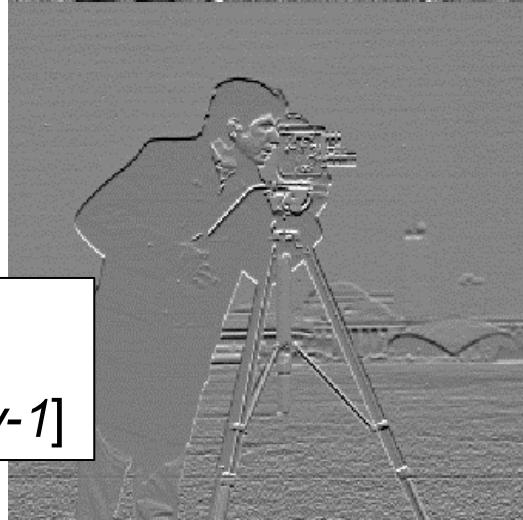
$s[x,y]$



$u_H[x,y] =$
 $s[x,y] - 0.95 s[x-1,y]$



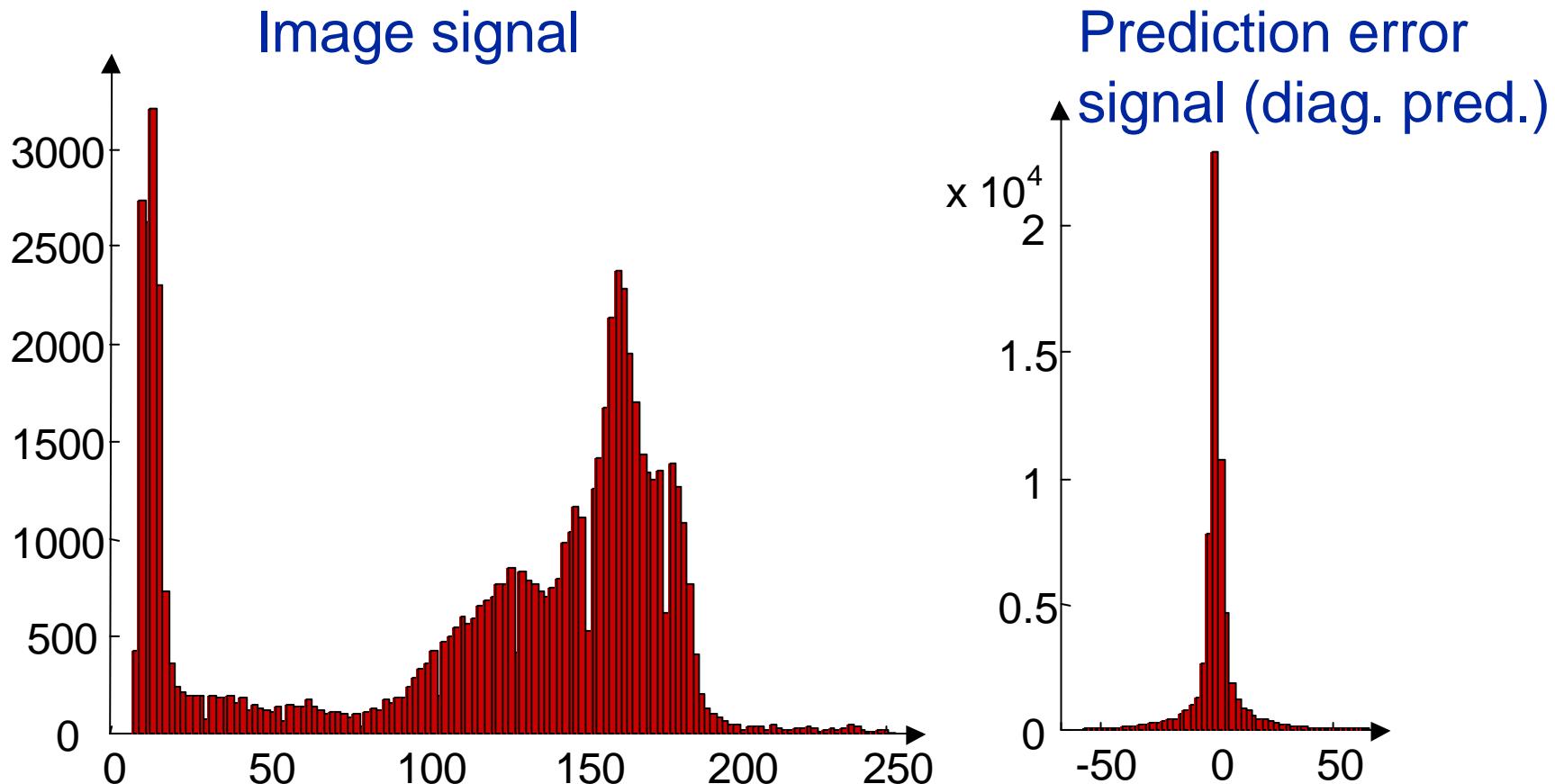
$u_V[x,y] =$
 $s[x,y] - 0.95 s[x,y-1]$



$u_D[x,y] = s[x,y] -$
 $0.5(s[x,y-1] + s[x-1,y])$



Change of Histograms: Cameraman



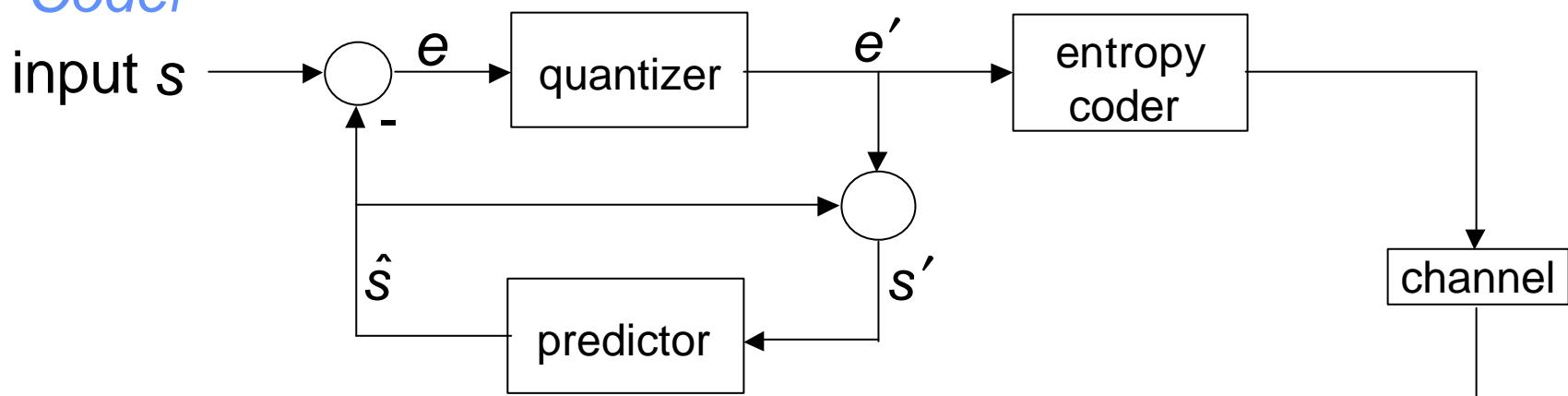
Can we use prediction for compression ?

Yes, if we reproduce the prediction signal at the decoder

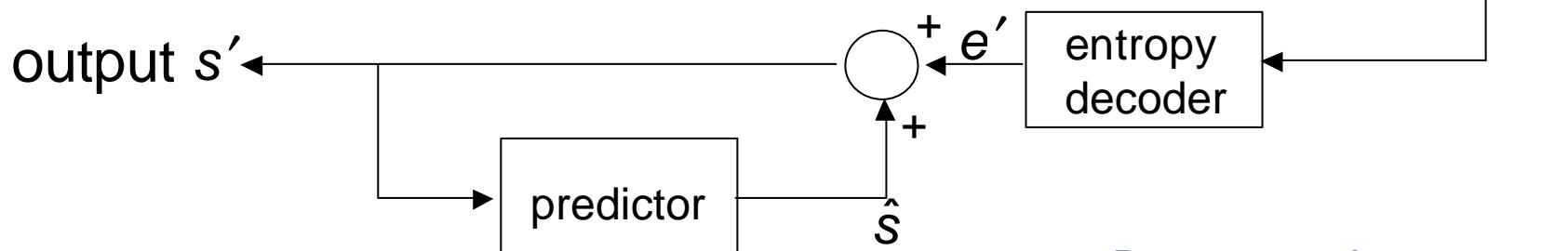


Differential Pulse Code Modulation

Coder



Decoder



Prediction error

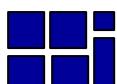
$$e = s - \hat{s}$$

Reconstruction

$$s' = e' + \hat{s}$$

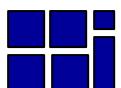
Reconstruction error =
quantization error

$$s' - s = e' - e = q$$



DPCM and Quantization

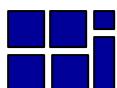
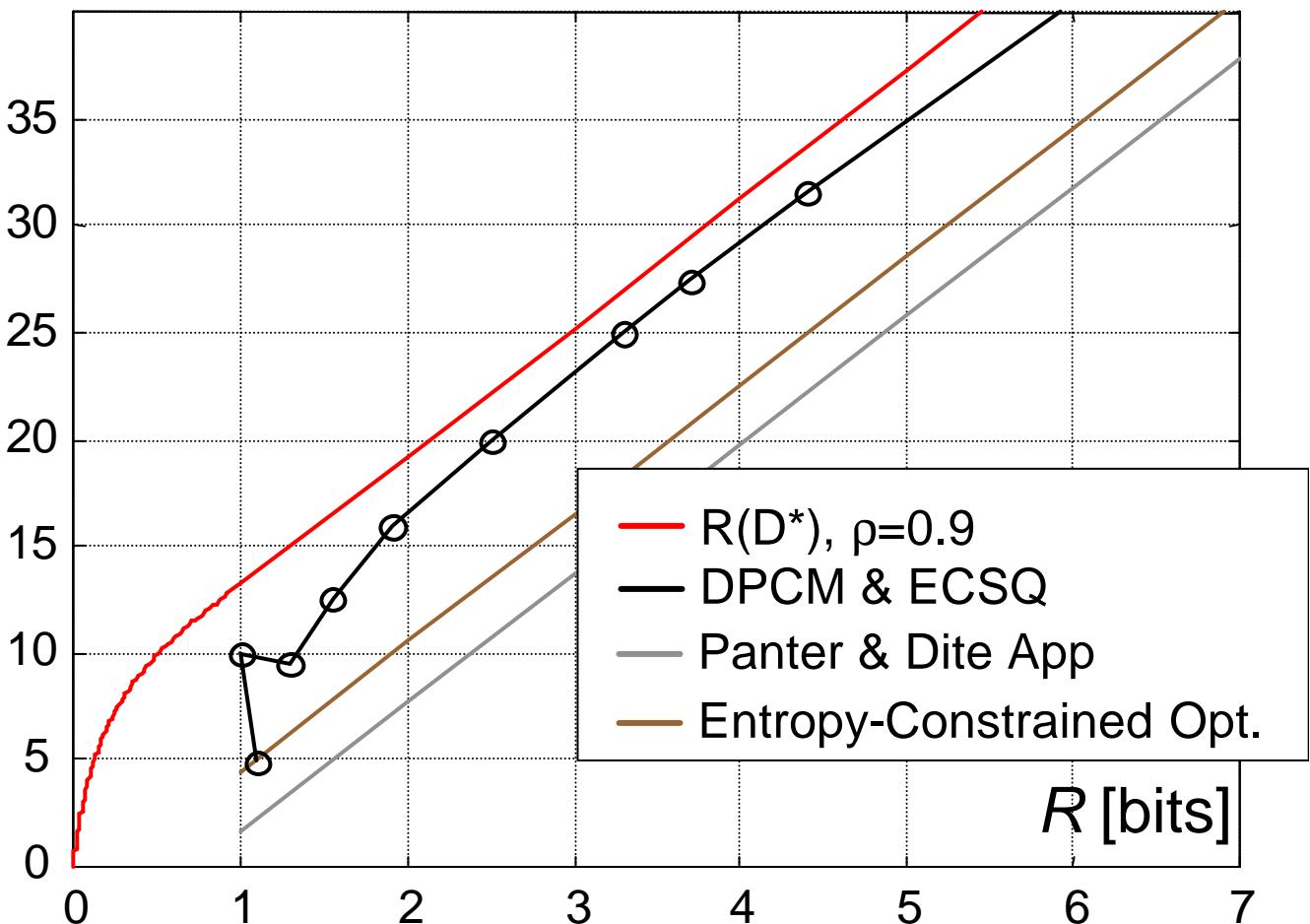
- Prediction is based on quantized samples
- Stability problems for large quantization errors
- Prediction shapes error signal (typical pdfs: Laplacian, generalized Gaussian)
- Simple and efficient: combine with entropy-constrained scalar quantization
- Higher gains: Combine with block entropy coding
- Use a switched predictor
 - Forward adaptation (side information)
 - Backward adaptation (error resilience, accuracy)
- DPCM can also be conducted for vectors
 - Predict vectors (with side information)
 - Quantize prediction error vectors



Comparison for Gauss-Markov Source: $\rho=0.9$

$$\text{SNR [dB]} = 10 \log_{10} \frac{s^2}{D}$$

- Linear predictor order $N=1, a=0.9$
- Entropy-Constrained Scalar Quantizer with Huffman VLC
- Iterative design algorithm applied



DPCM with entropy-constrained scalar quantization

Example: Lena, 8 b/p



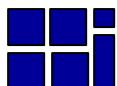
$K=511, H=4.79 \text{ b/p}$ $K=15, H=1.98 \text{ b/p}$ $K=3, H=0.88 \text{ b/p}$
K..number of reconstruction levels, H..entropy



Transmission Errors in a DPCM System

- For a linear DPCM decoder, the transmission error response is superimposed to the reconstructed signal S'
- For a stable DPCM decoder, the transmission error response decays
- Finite word-length effects in the decoder can lead to residual errors that do not decay (e.g., limit cycles)

from: Girod



Transmission Errors in a DPCM System II

Example: Lena, 3 b/p (fixed code word length)



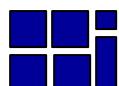
Error rate $p=10^{-3}$.

1D pred., hor. $a_H=0.95$

1D pred., ver. $a_V=0.95$

2D pred.* $,a_H=a_V=0.5$

from: Ohm



Inter-frame Coding of Video Signals

- Inter-frame coding exploits:
 - Similarity of temporally successive pictures
 - Temporal properties of human vision
- Important inter-frame coding methods:
 - Adaptive intra/inter-frame coding
 - Conditional replenishment
 - Motion-compensating prediction
 - Motion-compensating interpolation

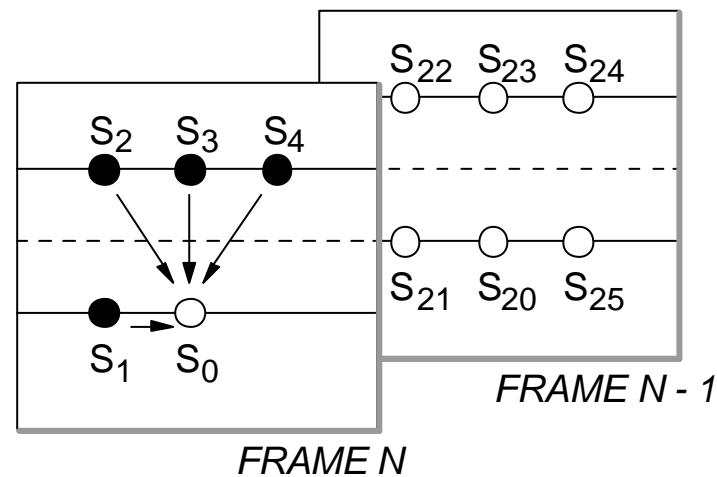
from: Girod



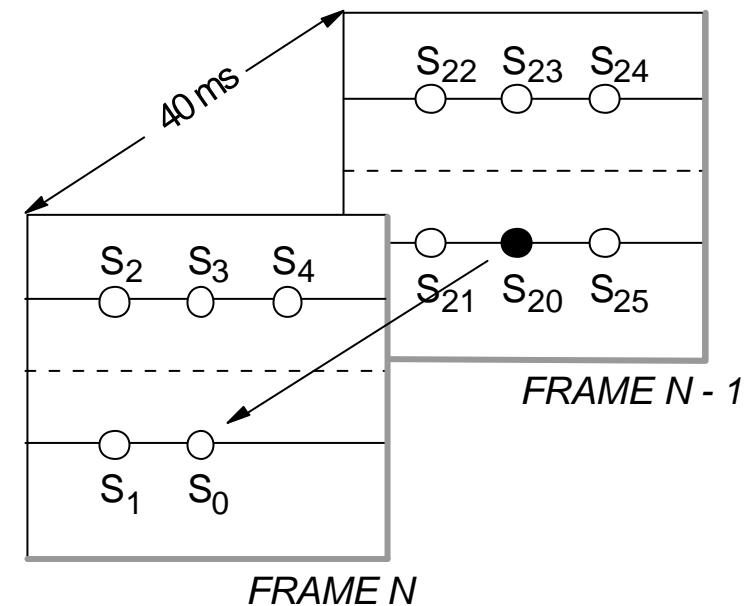
Principle of Adaptive Intra/Inter-frame DPCM

- Predictor is switched between two states:
for moving or changed areas.

Intra-frame prediction
for moving or changed areas.



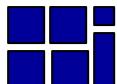
Inter-frame prediction (previous frame prediction) for still areas of the picture.



$$\hat{S} = a_1 \cdot S'_1 + a_2 \cdot S'_2 + a_3 \cdot S'_3 + a_4 \cdot S'_4$$

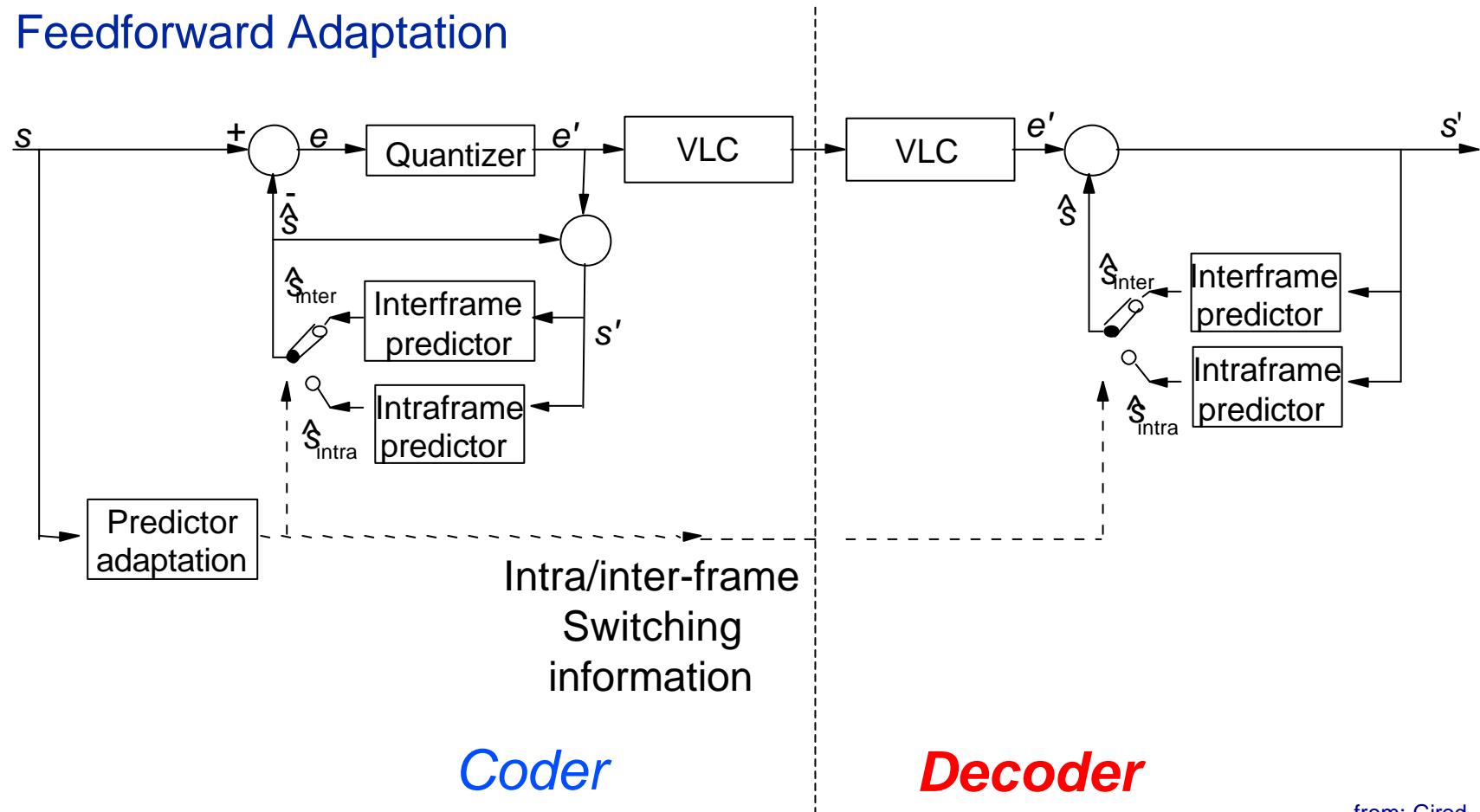
$$\hat{S}_{\text{inter}} = S'_{20}$$

from: Girod

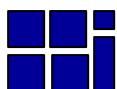


Intra/Inter-frame DPCM: Adaptation Strategies, I

Feedforward Adaptation

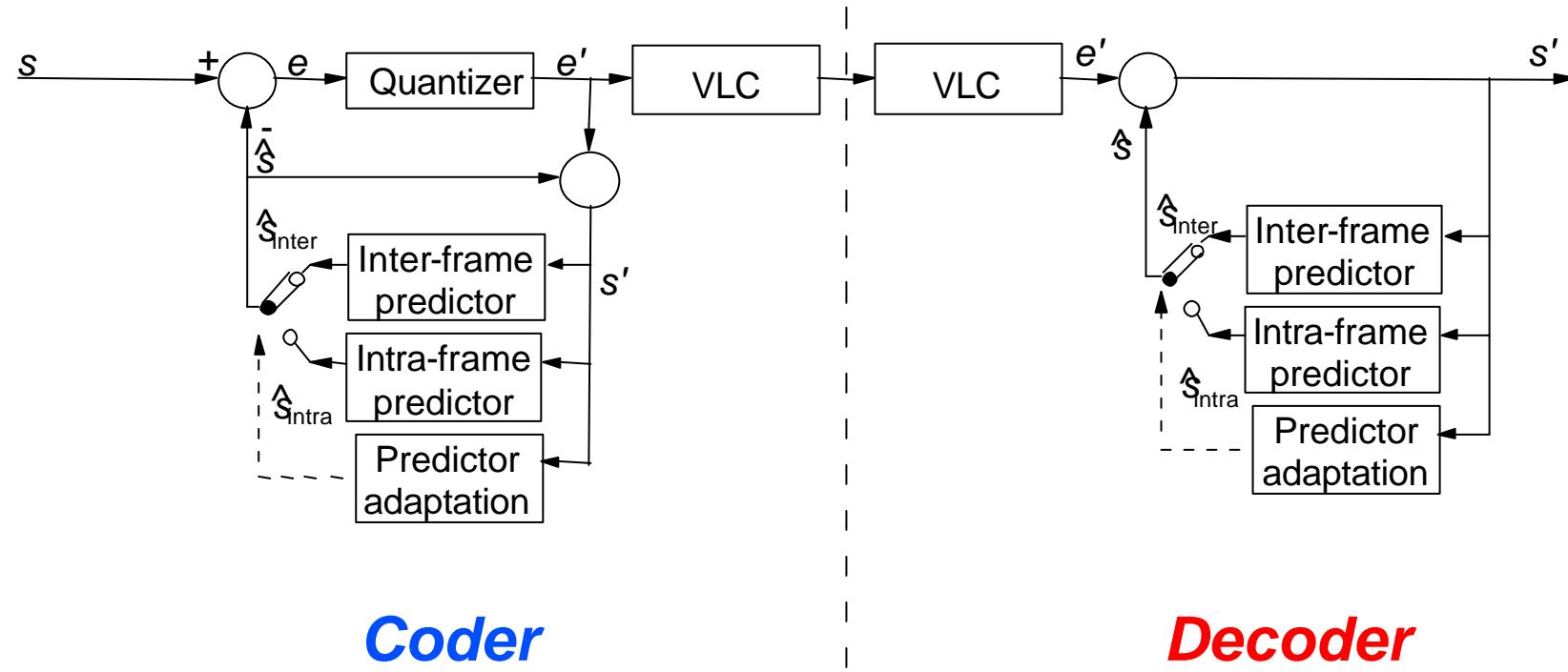


from: Girod

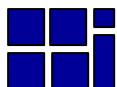


Intra/Inter-frame DPCM: Adaptation Strategies, II

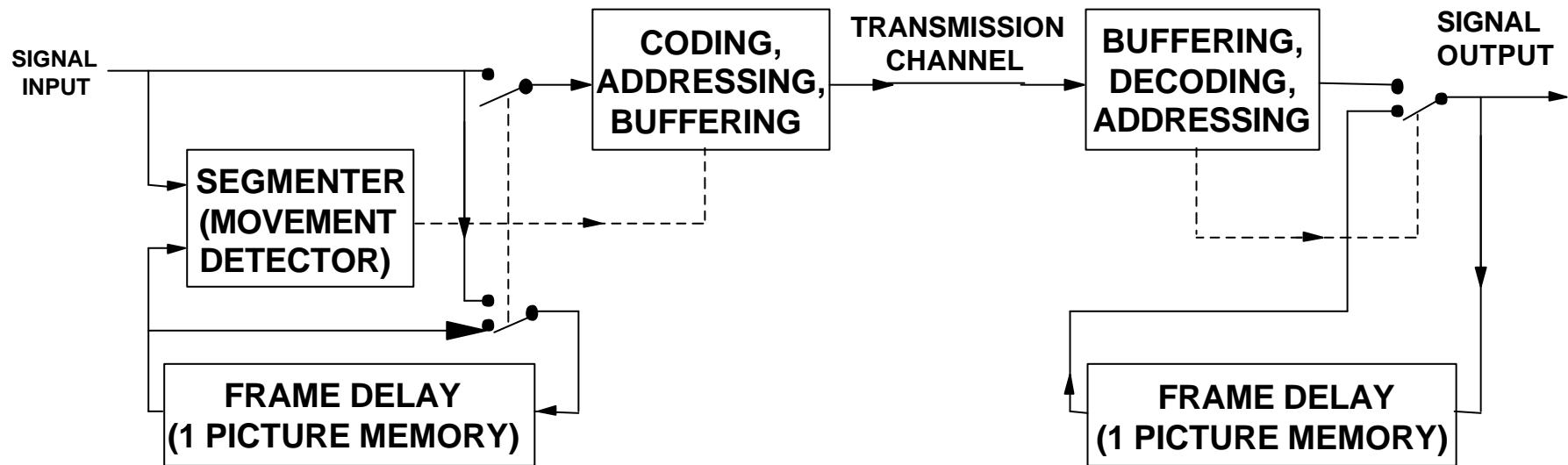
Feedback Adaptation



from: Girod



Principle of a Conditional Replenishment Coder

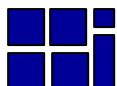


Coder

Decoder

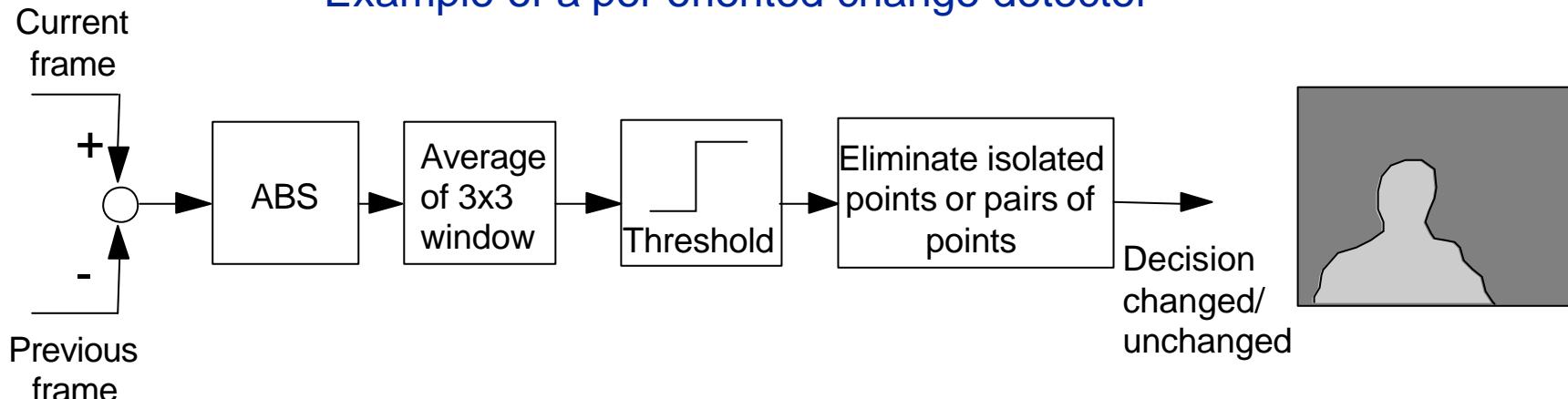
- Still areas: repeat from frame store
- Moving areas: transmit address and waveform

from: Girod

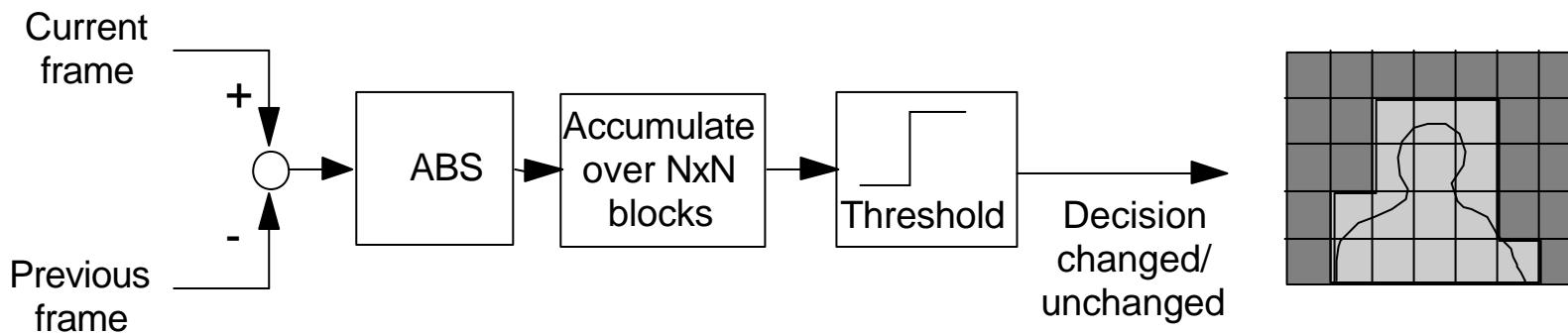


Change Detection

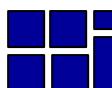
Example of a pel-oriented change detector



Example of a block-oriented change detector

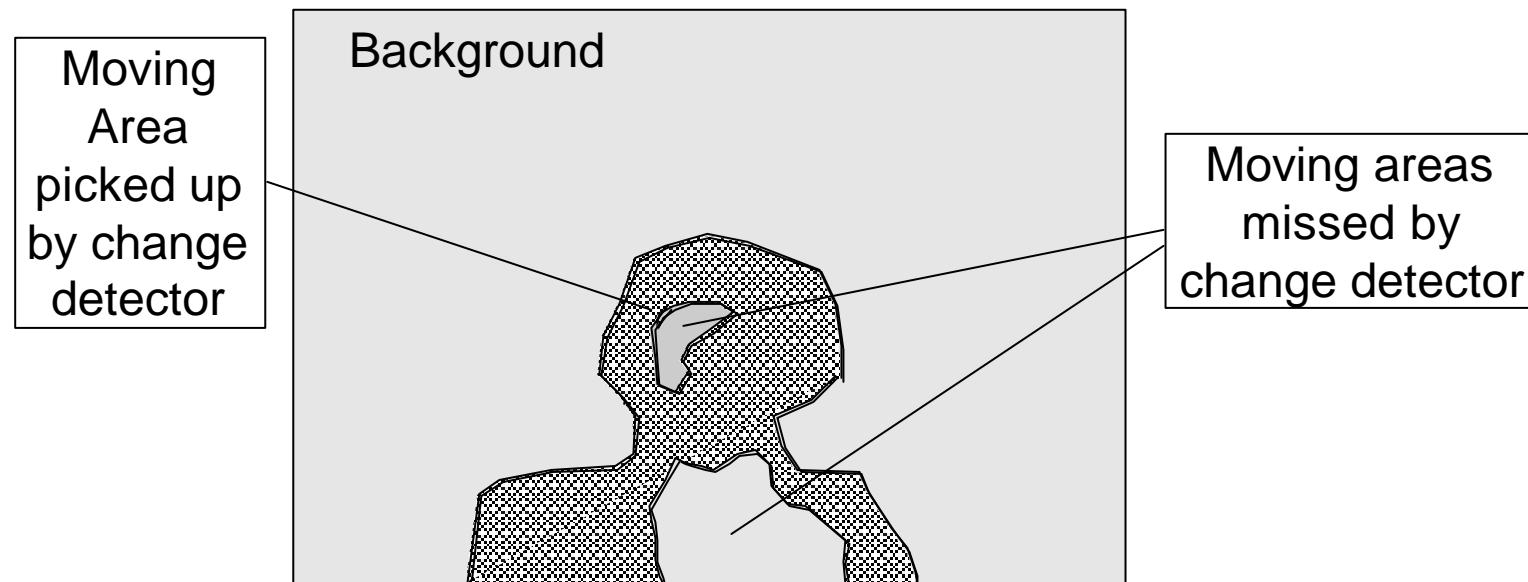


from: Girod

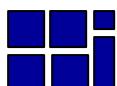


The "Dirty Window" Effect

- Conditional replenishment scheme with change detection threshold set too high leads to the subjective impression of looking through a dirty window.



from: Girod



Summary

- Prediction: Estimation of random variable from past or present observable random variables
- Optimal prediction
- Optimal linear prediction
- Prediction in images: 1-D vs. 2-D prediction
- DPCM: Prediction from previously coded/transmitted samples (known at coder and decoder)
- DPCM and quantization
- DPCM and transmission errors
- Adaptive Intra/Inter-frame DPCM: forward adaptation vs. backward adaptation
- Conditional Replenishment: Only changed areas of image are transmitted

